EXPERIMENTAL INVESTIGATION OF THE YIELD CONDITION IN RELATION TO DYNAMIC LOADING OF COMPRESSIBLE SOILS

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The yield function has been investigated in relation to the dynamic loading of various compressible soils (clays, loams, and loesses). The form of the yield function depends importantly on the composition and moisture content of the soils and on the loading conditions.

1. A rigid-plastic or elastoplastic material with a Prandtl–Mohr or Mises–Schleicher yield condition is often used as a model of a compressible soil. It is customary to employ a simplified interpretation in the form of linearized relations between the tangential and normal stresses [1-3]. At the same time it has been noted [2] that at sufficiently large dynamic loads a soil containing clay particles behaves like a fluid free of shear stresses.

In [4, 5] the role of viscous effects in the dynamic loading of various soils was established. It is legitimate to assume that viscous effects also influence the form of the yield function.

2. Our experiments were based on a method essentially the same as that described in [4]. The soil sample was contained in a cylinder and the load was applied by the impact of a falling weight on a moving piston. In order to vary the loading rate we employed elastic pads of variable thickness. We investigated clays and loams with natural structure and loess with disturbed structure. We tested two varities of Kerchenskii clays with density γ_0 = 1850 kg/m^3 and moisture content w = 23.5 and 15% by weight, Kiev loams (γ_0 = 1990 kg/m^3 and w = 14.17%), Kherson loams (γ_0 = 1830 kg/m^3 and w = 12.98%), and Kakhovskii loess (γ_0 = 1550 kg/m^3 and w = 7.02%).

The state of stress in the soil and its variation with time were measured with membrane strain gauges, whose readings were recorded on a N-700 loop oscillograph after preliminary amplification.

From an analysis of the oscillograms we determined (t is time) the principal components of the stress tensor

$$\sigma_1 = \sigma_z(t), \qquad \sigma_2 = \sigma_3 = \sigma_x(t) = \sigma_y(t)$$

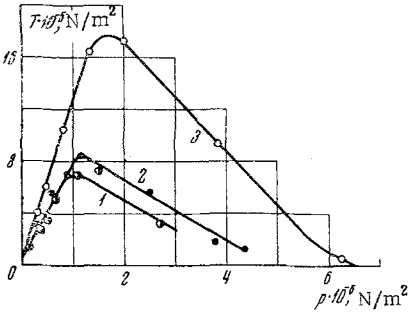

Fig. 1

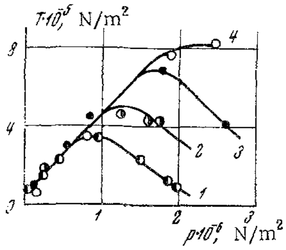

Fig. 2

Kiev. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 11, No. 2, pp. 131–133, March-April, 1970. Original article submitted May 14, 1969.

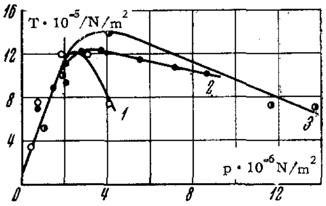

Fig. 3

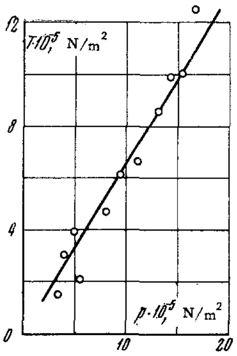

Fig. 4

the mean hydrostatic pressure in the soil

$$p = \frac{1}{3}[\sigma_z(t) + 2\sigma_x(t)] \tag{2.1}$$

and the shear stress intensity

$$T = \frac{1}{3}\sqrt{3}\,[\sigma_z(t) - \sigma_x(t)] \tag{2.2}$$

Eliminating the parameter t from (2.1) and (2.2), one can easily construct the yield condition $T = f(p)$.

The experimental $T = f(p)$ curves are presented in Figs. 1-4. Curves 1 and 2 in Fig. 1 correspond to clays with a moisture content of 23.5% at loading rates of $8.16 \cdot 10^8$ and $1.39 \cdot 10^9$ N/m^2 sec, respectively; curve 3 corresponds to the clay with a moisture content of 15% and a loading rate of $7.54 \cdot 10^8$ N/m^2 sec. The $T = f(p)$ curves for the Kiev loams are presented in Fig. 2; curve 1 corresponds to $\sigma = 3.07 \cdot 10^8$ N/m^2 sec; curve 2 to $\sigma = 3.2 \cdot 10^8$ N/m^2 sec; curve 3 to $\sigma = 4.64 \cdot 10^8$ N/m^2 sec, and curve 4 to $\sigma = 7.95 \cdot 10^8$ N/m^2 sec.

The corresponding relations for the Kherson loams are plotted in Fig. 3 (curves 1, 2, and 3 correspond to $\sigma = 2.42 \cdot 10^9$, $4.39 \cdot 10^9$, and $7.55 \cdot 10^9$ N/m^2 sec, respectively) and those for the loesses in Fig. 4.

3. An analysis of the results shows that the general form of the yield function for compressible soils is complex in character. At first, as the mean hydrostatic pressure increases, the shear stress intensity rises. At a certain pressure level the shear stresses reaches a maximum. Further loading is accompanied by a decrease in the shear stresses in the soil until it goes over into the compressible fluid state, when $T = 0$.

In general form, the $T = f(p)$ curves are satisfactorily approximated by the expression

$$T = a(p + p_0)^b \exp[c(p + p_0)] \tag{3.1}$$

Here, a, b, and c are empirical coefficients, and p_0 is a coefficient having the dimension of stress (see Fig. 5).

Using a yield condition in the form (3.1) leads to serious complication of the mathematical solutions, which is not always convenient from the practical standpoint. Therefore, in order to achieve simplification, in the first approximation the actual yield function can be arbitrarily represented in piecewise-linear form (Fig. 5):

$$T = \alpha + \beta p \quad \text{at} \quad 0 \leqslant p \leqslant p_* \tag{3.2}$$

$$T = \gamma - \delta p \quad \text{at} \quad p_* \leqslant p \leqslant p_b \tag{3.3}$$

$$T = 0 \quad \text{at} \quad p \geqslant p_b \tag{3.4}$$

Here, p_* is the pressure corresponding to the maximum shear stress in the soil; p_b is the pressure at which the shear stresses in the soil become negligibly small.

The coefficients α, β, γ, and δ are determined empirically. A yield function in the form (3.2) was first proposed by S. S. Grigoryan in applying the Mises–Schleicher yield condition to soils. Not being nondecreasing, this function retains its form on the interval of falling shear stresses. We note that, in form, expression (3.2) recalls the Coulomb law for an octahedral area element; accordingly, correct to the constants, the coefficients α and β may be regarded as coefficients characterizing the cohesion and internal friction in the soil, respectively.

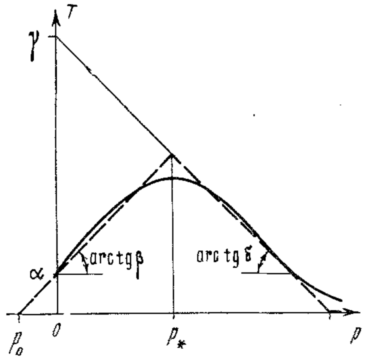

Fig. 5

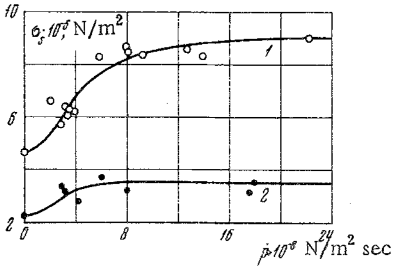

Fig. 6

4. An examination of the curves in Figs. 1-4 reveals that the described form of the yield function is considerably affected by the composition and moisture content of the soil. The higher the moisture content, the lower the values reached by the maximum shear stresses, and the lower the pressure at which the soil goes over into the compressible fluid state. This is in good agreement with ordinary engineering practice. At the same time, a reduction in the moisture content of the soil raises its strength and extends the limit of applicability of the yield condition in the form (3.2). This is confirmed both by Fig. 4 and by similar investigations made on soils with a low moisture content by other authors [4].

It should be noted that as the loading rate in the pressure pulse increases there is an increase in the maximum shear stresses in the soil, which suggests an increase in the resistance to deformation. This effect, observed in relation to almost all the soils, indicates the dependence of the yield condition on the so-called time parameters of the pressure pulse and is one of the manifestations of the viscous properties of soils associated with deformation.

The relative inaccuracy with which low pressures were recorded made it impossible to establish the effect of the loading rate on the form of the function $f(p)$ in the region $0 \leq p \leq p_*$. The existence of such an effect is confirmed by the established dependence of the ultimate strength of the soil, determined by cohesion and internal friction, on the loading rate presented for certain of the above-mentioned soils in Fig. 6 (curve 1 – clay with w = 23.5%, curve 2 – Kiev loam).

It follows that the coefficients a, b, and c, which determine the nature of the yield function, depend, in their turn, both on the physicomechanical properties of the soils and on the time parameters of the pressure pulse. Establishing these relationships will require special investigations. Here we content ourselves by merely noting their existence, as follows from an analysis of the curves presented in Fig. 1-3.

LITERATURE CITED

1. S. S. Grigoryan, "Fundamental concepts of soil dynamics," PMM, 24, No. 6, 1960.
2. Kh. A. Rakhmatulin, A. Ya. Sagomonyan, and N. A. Alekseev, Problems of Soil Dynamics [in Russian], Izd-vo MGU, Moscow, 1963.
3. S. S. Grigoryan, G. M. Lyakhov, V. V. Mel'nikov, and G. V. Rykov, "Blast waves in loess soils," PMTF, No.4, 1963.
4. V. V. Mel'nikov and G. V. Rykov, "Effect of deformation rate on the compressibility of loess soil," PMTF, No.2, 1965.
5. A. V. Mikhalyuk, "Determination of the size of cavities and compaction zones associated with explosions with allowance for their viscous properties," in: Explosion in Porous and Disperse Media [in Russian], Naukova Dumka, Kiev, 1969.